The effect of partial ionisation on the properties of MHD waves

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Introduction

The temperature in many plasma regions in space is not high enough to ensure a full ionisation, meaning that plasmas are made up of a mixture of positive ions, electrons and neutrals interacting though binary collision. In the context of solar physics the plasma can be considered to be partially ionised in the lower part of the atmosphere covering the whole photosphere and most of the chromosphere (including prominences) up to temperatures of about $10^4 K$. The key interaction in such plasmas is the collision between ions and neutrals, as this ensures an effective momentum transfer and coupling between different species. The modelling framework of the dynamics in such plasmas depends on the frequency domain in which we are interested. For frequencies larger than the ion-neutral collisional frequency, the plasma dynamics is studied within the two-fluid approximation, where separate equations will describe the evolution of charged particles and neutrals.

Motivation and Model

- We assume a gravitationally stratified plasma where equilibrium physical parameters depend on height (z).
- We suppose an isothermal plasma, so density and pressure vary with height with identical form.
- Due to the decrease of pressure with height, the flux tube will expand with height and we consider the equilibrium magnetic field to be $\sim e^{-z/2H}$.
- Now the sound/Alfvén speeds are constant.

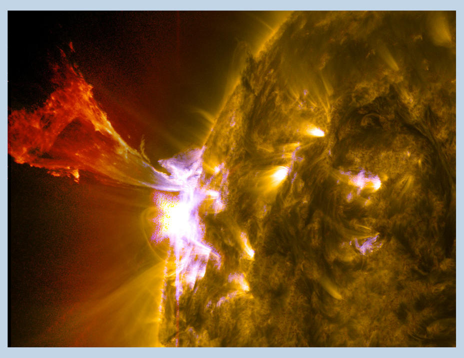

Figure 1: Burst of solar material leaps off the left side of the Sun in what's known as prominence eruption (Credit:NASA/SDO/AIA)

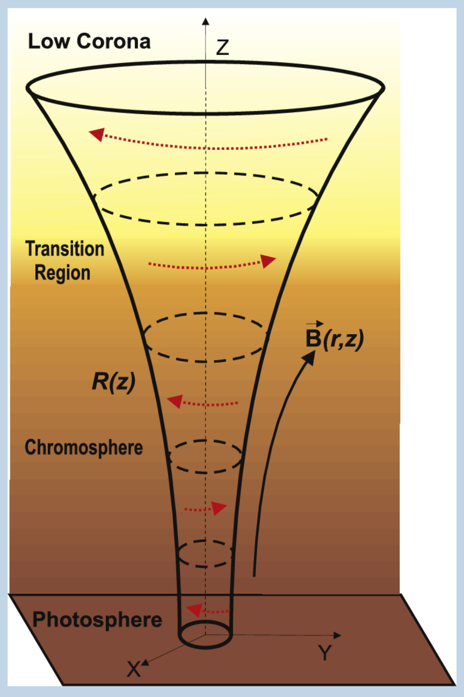

Figure 2: The expanding magnetic flux tube model embedded in the stratified atmosphere.

Governing Equations

In two-fluid (charges and neutral) plasmas the linearised governing equations for field-aligned slow magneto-acoustic waves are

$$\frac{\partial}{\partial t}(\rho_{0\alpha}A+\rho_\alpha A_0)+\frac{\partial}{\partial z}(\rho_{0\alpha}A_0 v_\alpha)=0, \alpha=i,n$$

$$\rho_{0\alpha}\frac{\partial v_\alpha}{\partial t}+\frac{\partial p_\alpha}{\partial z}+\rho_\alpha g \pm \rho_{0\alpha}\nu_{in}\mathcal{V}=0, \mathcal{V}=(v_i-v_n),$$

$$\frac{\partial p_\alpha}{\partial t}+v_\alpha\frac{dp_{0\alpha}}{dz}=c_{S\alpha}^2\left(\frac{\partial \rho_\alpha}{\partial t}+v_\alpha\frac{\partial \rho_{0\alpha}}{\partial z}\right),$$

together with the continuity of flux/total pressure

$$B_0 A+BA_0=0, \quad p_i+p_n+\frac{B_0}{\mu}B=\pi(z,t).$$

Here:
i,n stand for charged and neutral species.
ν_{in} is ion-neutral collision frequency,
A is the cross-section of the tube,
ρ and p are the density and pressure,
v_α is the speed of the species,
$c_{S\alpha}=(\gamma p_{0\alpha}/\rho_{0\alpha})^{1/2}$ is the sound speed of the species and γ is ratio of specific heats,
B the magnetic field and,
π_e the external pressure.

-Thermalisation of the plasma occurs very fast, over time comparable with collisional time.

Method and Assumptions

- Such plasma allow the propagation of two slow waves, one attached to each species. The evolution of waves is given by a system of two coupled Klein-Gordon (KG) equations.
- We solve the initial value problem for the coupled KG equations using Laplace transform in the strongly ionised limit.
- In this limit ion-acoustic modes propagate with no modification, only neutral-acoustic modes are affected by ions .
-The term describing the collision between particles will give rise to branch cuts in the complex plane, leading to wave attenuation.
-The boundaries of the flux tube are considered to be rigid, i.e. their temporal changes are very slow.

Results: The strongly ionised limit

- The spatio-temporal evolution of the two waves is given by a system of coupled KG equations of the form[2,3]

$$\hat{K}_i Q_i=0, \quad \hat{K}_i=\frac{\partial^2}{\partial t^2}-c_T^2\frac{\partial^2}{\partial z^2}+\omega_i^2. \tag{1}$$

$$\hat{K}_n Q_n=f(Q_i), \quad \hat{K}_n=\frac{\partial^2}{\partial t^2}-c_n^2\frac{\partial^2}{\partial z^2}+\omega_n^2. \tag{2}$$

- Here $\hat{K}_\alpha$ are the KG operators for the species α, c_T and c_n are the propagation speed of waves, ω_α are the cut-off frequencies for the two species waves can travel if their frequency is larger than the cut-off value.
-We drive the system (both species) with a harmonic pulse $\mathcal{A}_0(t)=V_0e^{-i\omega t}[H(t)-H(t-P)]$, where $P=2\pi/\omega$. This driver acts for a duration P, after which is stopped. The driver acts at $z=0$.
- The neutral-acoustic mode has a larger amplitude and decays slower than the corresponding ion-acoustic modes is shown in Figure 3.
- Only slow sausage waves associated with ions propagate with cut-off. The atmosphere acts as frequency filter for waves.
-Neutral acoustic modes propagate if their wavenumber satisfies the condition $k>\nu_{ni}/2c_{Sn}$.
- In strongly ionised plasmas these waves will have a very rapid decay, even in the absence of the simplifications $\longrightarrow$ any possible observation of these waves has to be carried out in an environment where the ionisation degree is moderate.

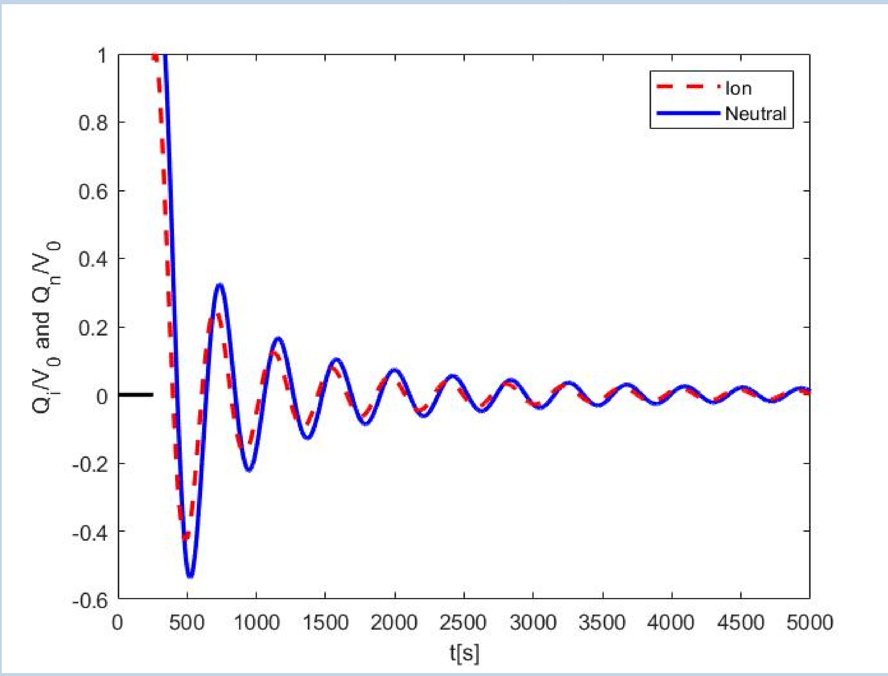

Figure 3: The temporal evolution of neutral-acoustic (solid lines) and ion-acoustic (dashed lines) modes at $z=4$ Mm (Alharbi et al. MNRAS, 2021.)

Conclusions and Future Studies

- Due to the stratified plasma both modes will have cut-off values, with the neutral cut-off always larger. The atmosphere acts as frequency filter for waves.
- The cut-off frequency of ion-acoustic waves is always larger than frequency of neutral-acoustic waves and they are weakly dependent on the value of plasma-β.
- The present analysis will be extended to a weakly ionised plasmas, fast waves and to plasmas in ionisation non-equilibrium.

References

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